Transactions of the VŠB – Technical University of Ostrava, Mechanical Series
No. 2, 2018, vol. LXIV
article No. 2044

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AN INTEGRATED MATHEMATICAL MODEL OF CREW SCHEDULING

INTEGROVANÝ MATEMATICKÝ MODEL PRO PLÁNOVÁNÍ LETOVÝCH POSÁDEK

Abstract

In conditions of air transport companies, the process of planning flight schedules is one of the most important processes each airline has to deal with. The flight schedule planning process consists of several consecutive plans. The first step of the planning process is defining which air routes will be operated, the decision is based on the business plan of the air transport company. Consequently, suitable airplanes have to be assigned to the individual air routes. And finally, on the basis of the previous steps shifts of pilots can be planned, the shifts are usually planned one month in advance.

However, with no respect to the created plan some unexpected disruptions of the flying staff, especially of the pilots, may happen in practice due to many reasons. In such cases the original plan has to be modified in order to react to the disruptions. The modifications can represent an optimisation problem – the air transport company has a set of the pilots and on the basis of their qualification and experience the company has to create new aircrews. The pilots can be found in different localities that are different from the airports of the planned flight departures. That means the newly planned aircrews are assigned to the individual flights with respect to costs associated with transportation of the aircrews to the airports of their departure.

The problem can be solved by many approaches. One of the possible approaches is a heuristic approach which is based on sequential solving two linear mathematical models. The first model decides about the aircrews (matches the pilots with respect to their compatibility). The second model solves the assignment problem – the air-crews are matched with the individual flights. The article presents an integrated linear model which deals with both problems at the same time.

Abstrakt

V podmínkách leteckých dopravců je hlavním výsledkem plánovacího procesu letový řád. Samotná tvorba letového řádu je posloupností několika na sebe navazujících dílčích plánů. Prvním krokem v procesu plánování je naplánování linek podle obchodního záměru dopravce, následně se naplánovaným letům přidělí konkrétní typ letadla. Zpravidla s měsíčním předstihem je nutné vytvořit plán práce pro posádky pilotů, kteří budou letouny obsluhovat.

Bez ohledu na vytvořený plán práce posádek však může dojít k neočekávaným výpadkům personálu. Potom je nutné operativně upravit připravený plán a posádky přeplánovat. Jedná se tedy o optimalizační problém, kdy dopravce má k dispozici množinu pilotů, z nichž je nutné na základě jejich kvalifikace a zkušeností vytvořit nové posádky. Piloti se mohou nacházet v různých destinacích, které mohou být různé od letišť odletů. Nově vytvořené posádky jsou potom přidělovány konkrétním letadlům v závislosti na velikosti nákladů spojených s přepravou posádek k letadlům.

Uvedený problém lze řešit různými způsoby. První způsob je heuristický založený na postupném řešení dvou lineárních modelů. V prvním modelu se rozhoduje o vytvoření posádek.

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Druhý model vytvořené posádky přiděluje letadlům. Cílem tohoto příspěvku bude prezentovat integrovaný lineární model řešící oba problémy současně.

Keywords

Mathematical modeling, Air transport, Linear programming.

1 INTRODUCTION

One of the main targets of each air transport company is to provide services of corresponding quality, especially with respect to fluency of the transportation process and delay minimisation, to its customers. On the other hand, because of strong competition each air transport company wants to minimise its costs. One of possible ways how to reach requested quality standards of the offered services with minimal costs is optimal planning of flight schedules and all the processes connected with them. The planning process of the air transport company is a chain consisting of several partial planning steps.

Within the first step it is necessary to create the flight schedule (a plan of the individual flights operated by the air transport company) on the basis of destinations which the company wants to offer. Consequently, the company must define arrival and departure times of all the flights which have been defined within the first step of the planning process. The arrival and departure times are influenced by operational conditions of airports above all. We can mention for example service hours of the airport, free time slots for take-off and landing or available technical equipment for aircraft ground handling.

The next step of the planning process is assigning suitable airplanes to the individual flights.

The assignment is also influenced by some factors, the most important are operational and technical parameters of the airplanes such as their range of flight, capacity and so on. Another important criterion which should be considered is an expected demand for transportation on the air route or technical parameters of the airports. If the air transport company operates a homogenous fleet of airplanes, then this part of the planning process is simplified essentially.

After matching the available airplanes with the planned flights, it is necessary to plan crews (pairs of pilots) for the flights. The crew planning process can be divided into two partial steps. Within the first step the pilots are matched, that means we create the pairs of them. After it we can assign the created crews to the individual flights. Both planning processes can be successfully solved by means of mathematical programming methods. Using the methods may simplify the decision process and we obtain an optimal solution when applying the methods correctly.

2 CREW PAIRING

Pilot matching is a complex process and it is influenced by many factors. In other words, many reasons why two pilots can be or cannot be matched may be given in reality. One of the most important factors is the fact that each crew has to be formed by a captain (a pilot in command) and a first officer. That means we cannot create any crew consisting of the pilots who do not have the rank of captain – the pilots are not qualified to be the pilot in command. Another influencing factor is seniority of the pilots. For our purposes we can divide the pilots into the experienced pilots and inexperienced ones on the basis of their level of experience. When pairing the pilots, we should prefer the crews consisting of an experienced pilot and an inexperienced one.

Another factor which can influence the process of matching the pilots is dividing the pilots into instructors and cadet pilots (under training). In such cases the instructor pilot has to be matched with the cadet pilot. An-other factor which should be mentioned is dividing airports into three classes according to difficulty of take-off and landing – three categories A, B and C are defined, where the class C is the most difficult category of the airports. Each pilot has to be qualified in order to land at the airports of the individual categories. That means a pilot who is qualified only for the category A must not land at any airport of the category B or C.

Other reasons for matching or not matching the pilots can be represented by different personal or family reasons. Two pilots must not be matched it they do not like one another because such aversion may bring potentially dangerous situations during the whole flight. Analogously, in case of married couples we try not to match them. From the psychological point of view it is usual that one person is dominant in the married couples and this fact may negatively influence decisions, especially in case of emergency, of the pilot in command if he/she is the person who is not dominant in the marriage.

However, with no respect to the created plan some unexpected disruptions of the flying staff, especially of the pilots, may happen in practice due to many reasons. In such cases the original plan has to be modified in order to react to the disruptions. The modifications can represent an optimisation problem – the air transport company has a set of the pilots and on the basis of their qualification and experience the company has to create new aircrews. The pilots can be found in different localities that are different from the airports of the planned flight departures. That means the newly planned aircrews are assigned to the individual flights with respect to costs associated with transportation of the aircrews to the airports of their departure.

3 STATE OF ART

The presented article follows in article [1] in which a heuristic method for crew planning and consequent assigning of the crews to individual flights is demonstrated. The method consists of two linear mathematical models. The first model matches the available pilots to create crews and the second one assigns the crews to the individual flights.

A heuristic method was developed by authors of article [2], the authors focus on assigning the crews to the individual flights. The experiments with the proposed method revealed that the method can be applied for planning up to 27 000 flights. An optimization criterion which is used in the heuristic corresponds to the total costs of assigning the crews to the flights. The developed method was applied on real examples. Results obtained by the method were compared with the planning process of the air transport company Air France in article [3]. The comparison revealed that applying the heuristic brings potential savings in planning the crews.

Another practical application is presented in article [4] in which the crew planning process in conditions of Taiwan Airlines was improved by means of a network model. In articles [5] and [6] authors focused on connecting the crew planning process with assigning the crews to individual airplanes. In article [7] authors create the crews with respect to uniformly distributed shifts of all the pilots. The authors present a two-step algorithm which matches the pilots to create the crews and generates their rest times.

An interesting approach to crew planning was used in article [8]. Authors of the article defined constraints which result from natural human body cycles; they incorporated the constraints in a mathematical model. They point out that even when fatigue of the pilots from previous flights is considered in the crew planning process, so often ordered breaks are not enough to eliminate their fatigue.

In publication [9] the crew planning problem with time coordination of the flights is integrated. To solve the integrated problem a combination of several methods – such as column generation method or generating dynamic constraints – was applied. An application of the proposed method brought a decrease in need for the airplanes and their crews.

Article [2] deals with operational planning of the crews in case of disruptions of the planned crews. Authors try to minimise financial losses due to flight delays or cancelling some flights. A proposed algorithm identifies flights which may bring the biggest losses in case of delaying or cancelling them. For these flights the algorithm searches for an alternative solution

4 MATHEMATICAL FORMULATION

Let a set I of the pilots that can be assigned to the crew be given. Cardinality of the set I is $|I| = m$. Let also K represents a set of flights to crew assignment. Cardinality of the set K is $|K| = n$. Possible matching of pilots is modelled by an adjacency matrix a_{ij}, where $i \in I$ and $j \in I$. The adjacency matrix a_{ij} contains binary values. If $a_{ij} = 1$, then the pilot $i \in I$ can be matched with the pilot $j \in I$. If $a_{ij} = 0$, then the pilot $i \in I$ cannot be matched with the pilot $j \in I$. It has to be assured that $i \neq j$, because the crew must consist of two different pilots. This condition will be ensured by the value $a_{ij} = 0$ on the main diagonal of the adjacency matrix (for $i = j$).

Moreover a matrix of costs d_{ik} must be defined. Elements of the matrix correspond to the costs of assigning the pilot $i \in I$ to the flight $k \in K$ (for example the travel costs if the pilot is not present at an airport where the flight starts). The matrix of the costs d_{ik} is identical to a matrix of the costs d_{jk}, that corresponds to the costs of assigning the pilot $j \in I$ to the flight $k \in K$.

A group of binary variables x_{ijk} modelling matching the corresponding pilots $i \in I$ and $j \in I$ with simultaneous assignment to one of the flights $k \in K$ is used in the model. Our task is to minimise the costs of assigning the matched pilots $i \in I$ and $j \in I$ to the flights $k \in K$. The model can be defined as follows:

$$\min f(x) = \sum_{i \in I} \sum_{j \in I} \sum_{k \in K} (d_{ik} + d_{jk}) \cdot x_{ijk} \tag{1}$$

subject to:

$$\sum_{i \in I} \sum_{j \in I} a_{ij} \cdot x_{ijk} = 1 \text{ for } k \in K, \tag{2}$$

$$\sum_{j \in I} \sum_{k \in K} (a_{ij} \cdot x_{ijk} + a_{ji} \cdot x_{jik}) \leq 1 \text{ for } i \in I, \tag{3}$$

$$x_{ijk} \in \{0,1\} \text{ for } i \in I, j \in I, k \in K. \tag{4}$$

Function (1) corresponds to the objective function of the model. The optimisation criterion represents minimisation of the costs of assigning the pilots to the flights. Constraint (2) is a group of equations for every flight $k \in K$. The constrains ensure that exactly one crew will be assigned to every flight $k \in K$. Constraints (3) ensure that pilot $i \in I$ will be assigned at most once to the flight $k \in K$. Constraints (4) define a domain of definition of the variables. Variable x_{ijk} is binary and it decides about crew composition and subsequent assignment to the flight $k \in K$. If $x_{ijk} = 1$, then the pilot $i \in I$ is matched with the pilot $j \in I$ and they are assigned to the flight $k \in K$. If $x_{ijk} = 0$, then the pilot $i \in I$ cannot be matched with the pilot $j \in I$ and/or the crew cannot be assigned to the flight $k \in K$.

4.1 Experiment with model

To test the mathematical model some experiments were carried out, in the article we present one of them. All the experiments were realised by means of optimisation software Xpress-IVE. Input data for the presented example are the following:

- The set I of the pilots that are available, where $|I| = 21$,
- The adjacency matrix a_{ij}, where $i \in I$ and $j \in I$,

- The set K of the flights to which a suitable crew must be assigned, where $|K| = 10$,
- The cost matrix, where $i \in I$ and $k \in K$.

$$a_{ij} = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0
\end{bmatrix}$$

$$d_{ik} = \begin{bmatrix}
7 & 8 & 5 & 6 & 4 & 5 & 3 & 9 & 4 & 5 \\
4 & 7 & 4 & 5 & 6 & 2 & 6 & 5 & 3 & 4 \\
5 & 6 & 8 & 4 & 2 & 4 & 5 & 6 & 3 & 2 \\
8 & 9 & 6 & 1 & 2 & 1 & 2 & 7 & 9 & 6 \\
2 & 6 & 5 & 3 & 4 & 2 & 3 & 4 & 6 & 4 \\
7 & 6 & 9 & 4 & 5 & 1 & 2 & 7 & 8 & 4 \\
4 & 5 & 6 & 8 & 2 & 3 & 6 & 5 & 2 & 4 \\
4 & 3 & 9 & 4 & 7 & 5 & 6 & 3 & 8 & 4 \\
4 & 6 & 5 & 2 & 8 & 4 & 5 & 6 & 3 & 9 \\
9 & 4 & 8 & 1 & 6 & 7 & 8 & 5 & 2 & 4 \\
4 & 8 & 5 & 6 & 2 & 1 & 2 & 8 & 3 & 6 \\
7 & 6 & 2 & 4 & 8 & 6 & 9 & 8 & 6 & 4 \\
6 & 5 & 8 & 4 & 2 & 5 & 7 & 9 & 3 & 1 \\
6 & 3 & 9 & 4 & 5 & 2 & 7 & 8 & 4 & 4 \\
5 & 5 & 6 & 6 & 9 & 9 & 2 & 2 & 3 & 8 \\
5 & 3 & 9 & 4 & 4 & 6 & 7 & 6 & 6 & 9 \\
1 & 2 & 1 & 2 & 1 & 4 & 5 & 6 & 7 & 1 \\
3 & 6 & 3 & 1 & 8 & 3 & 5 & 2 & 4 & 8 \\
5 & 9 & 6 & 3 & 8 & 4 & 2 & 5 & 6 & 8 \\
6 & 9 & 8 & 9 & 9 & 8 & 5 & 8 & 5 & 2 \\
1 & 4 & 5 & 8 & 6 & 8 & 6 & 4 & 8 & 6
\end{bmatrix}$$

After implementation of mathematical model (1) – (4) in optimisation software Xpress-IVE an optimal solution was found. The total costs of assigning the matched pilots to the flights are equal to 43 monetary units. Because we have 21 available pilots and 10 flights, it is necessary to create 10 crews and the remaining pilot is not matched – in case of our experiment pilot 16 is not matched. Results of the experiment are summarised in Table 1.

The first and the second columns of the table give information about matching the pilots; each pilot is represented by his/her number. In the third column the flights to which the individual crews are assigned are presented. The remaining columns summarise the costs associated with assigning the pilots $i \in I$ to the flights $k \in K$ - the values of d_{ik} in the fourth column, the costs associated with assigning the pilots $j \in I$ to the flights $k \in K$ - the values of d_{jk} in the fifth column and finally their sums in the last column. The sum of the values of $d_{ik} + d_{jk}$ in the last column gives a value of the optimisation criterion.

Table 1 The results of the optimisation experiment

pilot i	**pilot** j	**flight** k	d_{ik} **[*mu*]**	d_{jk} **[*mu*]**	$d_{ik} + d_{jk}$
5	21	1	2	1	3
14	8	2	3	3	6
9	12	3	5	2	7
19	10	4	3	1	4
17	11	5	1	2	3
2	4	6	2	1	3
6	1	7	2	3	5
18	15	8	2	2	4
3	7	9	3	2	5
13	20	10	1	2	3
					43 mu

5 CONCLUSION

Air transport cannot be operated without precise planning. In order to minimise economical losses incurred by delays of the flights, the air transport companies are forced to plan the flight schedules and all the processes connected with them conscientiously. Planning the flight plan consists of partial subsequent phases. In the first phase the air routes are chosen on the basis of the passengers' demand and offered destinations. Based on the air routes the individual flights have to be planned. For each flight a suitable airplane must be chosen.

The last part of the planning process is crew planning which comprises two tasks – matching the pilots (each crew consists of two pilots) and assigning the crews to the individual flights. The presented article dealt with the crew planning process. The linear mathematical model was presented in the article; the model matches the pilots based on their mutual compatibility which is expressed by the adjacency matrix and the matched pilots assigns to the flights that have to be dispatched based on the incurred costs

Functionality of the proposed model was tested on some examples; in the article one of the tested examples was presented. The results of the experiments indicate that the proposed mathematical model can solve the crew planning problem effectively

REFERENCES

[1] GRAF, V., TEICHMANN, D., DORDA, M. Contribution to crew rostering in air transport. In *Proceedings of the Quantitative Methods in Economics,* Slovak Society for Operation Resears, Trenčianské Teplice, 23.5. – 25.5.2018.

[2] DOI, T., NISHI, T., VOß, S. Two-level decomposition-based matheuristic for airline crew rostering problems with fair working time. *European Journal of Operational Research.* 2018, 267(2), pp. 428-438.

[3] ERDOGAN, G., HAOUARI, M., MATOGLU, M. O., OZENER, O. O. Solving a large-scale crew pairing problem. *Journal of the Operational Research Society*. 2015, 66(10), pp. 1742-1754.

[4] YAN, S., TU, Y-P. A network model for airline cabin crew scheduling. *European Journal for Operation Research.* 2002, 140(3), pp. 531-540.

[5] DEVECI, M., DEMIREL, N. C. Evolutionary algorithm for solving the airline crew pairing problem. *Computers & Industrial Engineering.* 2018, 115, pp. 389-406.

[6] QUESNEL, F., DESAULNIERS, G., SOUMIS, F. A new heuristic branching scheme for the crew pairing problem with base constraints. *Computers and Operational Research.* 2017, 80. pp. 159-172.

[7] YILDIZ, B. C., GZARA, F., ELHEDHLI, S. Airline crew pairing with fatigue: Modeling and analysis. *Transportation Research Part C.* 2017, 74, pp. 99-112.

[8] MERCIER, A., SOUMIS, F. An integrated aircraft routing, crew scheduling and flight retiming model. *Computers &Operations Research.* 2017, 34(8), pp.2251-2265.

[9] YEN, J.W., BIRGE, J.R. A stochastic programming approach to the airline crew scheduling problem. *Transportation science.* 2006, 40(1), pp. (3-14).